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**Practice Problems: Exercise 5 – Microengineering 110**

**Spring 2025**

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1. A new battery's voltage may be acceptable (A) or unacceptable (U). A certain flashlight requires two batteries, so batteries will be independently selected and tested until two acceptable ones have been found. Suppose that 90% of all batteries have acceptable voltages. Let  $Y$  denote the number of batteries that must be tested.
  - a. What is  $p(2)$ , that is,  $P(Y = 2)$ ?
  - b. What is  $p(3)$ ? [Hint: There are two different outcomes that result in  $Y = 3$ .]
  - c. To have  $Y = 5$ , what must be true of the fifth battery selected? List the four outcomes for which  $Y = 5$  and then determine  $p(5)$ .
  - d. Use the pattern in your answers for parts (a)–(c) to obtain a general formula for  $p(y)$ .
2. Two fair six-sided dice are tossed independently. Let  $M$  = the maximum of the two tosses (so  $M(1,5) = 5$ ,  $M(3,3) = 3$ , etc.).
  - a. What is the pmf of  $M$ ? [Hint: First determine  $p(1)$ , then  $p(2)$ , and so on.]
  - b. Determine the cdf of  $M$  and graph it.

3. The pmf of the amount of memory  $X$  (GB) in a purchased flash drive was given as

$x$	1	2	4	8	16
$p(x)$	.05	.10	.35	.40	.10

Compute the following:

a.  $E(X)$

b.  $V(X)$

- c. The standard deviation of  $X$

  

- 4. A company that produces fine glassware knows from experience that 10% of its goblets have cosmetic flaws and must be classified as “seconds.”
  - a. Among six randomly selected goblets, how likely is it that only one is a second?
  - b. Among six randomly selected goblets, what is the probability that at least two are seconds?
  - c. If goblets are examined one by one, what is the probability that at most five must be selected to find four that are not seconds?

  

- 5. Suppose that the number of drivers who travel between a particular origin and destination during a designated time period has a Poisson distribution with parameter  $\mu = 20$ . What is the probability that the number of drivers will
  - a. Be at most 10?
  - b. Exceed 20?
  - c. Be between 10 and 20, inclusive?

d. Be strictly between 10 and 20?

e. Be within 2 standard deviations of the mean value?

6. The current in a certain circuit as measured by an ammeter is a continuous random variable  $X$  with the following density function:

$$f(x) = \begin{cases} .075x + .2 & 3 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

a. Graph the pdf and verify that the total area under the density curve is indeed 1.

b. Calculate  $P(X \leq 4)$ . How does this probability compare to  $P(X < 4)$ ?

c. Calculate  $P(3.5 \leq X \leq 4.5)$  and also  $P(4.5 < X)$ .

7. Spray drift is a constant concern for pesticide applicators and agricultural producers. The inverse relationship between droplet size and drift potential is well known. The normal distribution with mean  $1050 \mu\text{m}$  and standard deviation  $150 \mu\text{m}$  was a reasonable model for droplet size for water (the “control treatment”) sprayed through a  $760 \text{ ml/min}$  nozzle.

- What is the probability that the size of a single droplet is less than  $1500 \mu\text{m}$ ? At least  $1000 \mu\text{m}$ ?
- What is the probability that the size of a single droplet is between  $1000$  and  $1500 \mu\text{m}$ ?
- If the sizes of five independently selected droplets are measured, what is the probability that exactly two of them exceed  $1500 \mu\text{m}$ ?