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Practice Problems: Exercise 5 – Microengineering 110

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1. A new battery's voltage may be acceptable (A) or unacceptable (U). A certain flashlight requires two batteries, so batteries will be independently selected and tested until two acceptable ones have been found. Suppose that 90% of all batteries have acceptable voltages. Let Y denote the number of batteries that must be tested.
 - a. What is $p(2)$, that is, $P(Y = 2)$?
 - b. What is $p(3)$? [Hint: There are two different outcomes that result in $Y = 3$.]
 - c. To have $Y = 5$, what must be true of the fifth battery selected? List the four outcomes for which $Y = 5$ and then determine $p(5)$.
 - d. Use the pattern in your answers for parts (a)–(c) to obtain a general formula for $p(y)$.
2. Two fair six-sided dice are tossed independently. Let M = the maximum of the two tosses (so $M(1,5) = 5$, $M(3,3) = 3$, etc.).
 - a. What is the pmf of M ? [Hint: First determine $p(1)$, then $p(2)$, and so on.]
 - b. Determine the cdf of M and graph it.

3. The pmf of the amount of memory X (GB) in a purchased flash drive was given as

x	1	2	4	8	16
$p(x)$.05	.10	.35	.40	.10

Compute the following:

a. $E(X)$

b. $V(X)$

- c. The standard deviation of X

- 4. A company that produces fine glassware knows from experience that 10% of its goblets have cosmetic flaws and must be classified as “seconds.”
 - a. Among six randomly selected goblets, how likely is it that only one is a second?

 - b. Among six randomly selected goblets, what is the probability that at least two are seconds?

 - c. If goblets are examined one by one, what is the probability that at most five must be selected to find four that are not seconds?

- 5. Suppose that the number of drivers who travel between a particular origin and destination during a designated time period has a Poisson distribution with parameter $\mu = 20$. What is the probability that the number of drivers will
 - a. Be at most 10?

 - b. Exceed 20?

 - c. Be between 10 and 20, inclusive?

- d. Be strictly between 10 and 20?
 - e. Be within 2 standard deviations of the mean value?
6. The current in a certain circuit as measured by an ammeter is a continuous random variable X with the following density function:

$$f(x) = \begin{cases} .075x + .2 & 3 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

- a. Graph the pdf and verify that the total area under the density curve is indeed 1.

- b. Calculate $P(X \leq 4)$. How does this probability compare to $P(X < 4)$?

- c. Calculate $P(3.5 \leq X \leq 4.5)$ and also $P(4.5 < X)$.

7. Spray drift is a constant concern for pesticide applicators and agricultural producers. The inverse relationship between droplet size and drift potential is well known. The normal distribution with mean $1050\text{ }\mu\text{m}$ and standard deviation $150\text{ }\mu\text{m}$ was a reasonable model for droplet size for water (the “control treatment”) sprayed through a 760 ml/min nozzle.
- a. What is the probability that the size of a single droplet is less than $1500\text{ }\mu\text{m}$? At least $1000\text{ }\mu\text{m}$?
 - b. What is the probability that the size of a single droplet is between 1000 and $1500\text{ }\mu\text{m}$?
 - c. If the sizes of five independently selected droplets are measured, what is the probability that exactly two of them exceed $1500\text{ }\mu\text{m}$?